5. H. Utida et al., Tokyo Daigaku Kogabuku Kiyo, A, No. 8 (1970).
6. G. F. Hewitt and N. S. Hall-Taylor, Annular Two-Phase Flows, Pergamon Press, Oxford—New York (1970).

GROWTH OF A BUBBLE OF NONCONDENSING GAS INJECTED INTO A LIQUID

B. N. Yudaev, O. V. Tsirlin,
and A. A. Yushkin

UDC 532.529

The solution of the problem of growth of a gas–vapor bubble injected into a liquid is obtained. The growth of the bubble depends on mass transfer in the gas phase and heat transfer in the liquid phase. Experimental and theoretical data are compared.

The solution of the problem of growth of a gas–vapor bubble is essential for investigation of the cooling of a liquid by bubbling gas through it and by other processes. Attempts to solve this problem are described in [1-3]. Neglecting the convective terms in the energy and diffusion equations, the authors of [1] found a solution of the problem of unsteady heat conduction with the aid of the Green function for the case of a stationary boundary. In [2, 3] a solution was obtained by assignment of the radial distribution of vapor concentration in the gas and the temperature in the liquid in the form of a quadratic parabola, which led to distortion of the values of the heat and mass fluxes. There are also errors in [3]. The listed solutions are approximate.

The main factors governing bubble growth are diffusion of vapor into the gas cavity and the redistribution of temperature in the liquid layer adjacent to the bubble. These factors act simultaneously, but, to begin with, it is better to consider them separately. It should be noted that since the thermal diffusivity in gases is greater than the diffusion coefficient, we can assume that heat transfer in the gas has no effect on diffusion.

We will solve the problem of vapor diffusion into a spherical cavity of prescribed variable radius R(t) with a prescribed variable concentration $x_s(t)$ on its boundary. Let the gas be insoluble and noncondensing, the pressure constant everywhere throughout the process, and the vapor in the cavity have nonzero velocity at the moment of entry. We have to solve the equation

$$\frac{\partial x}{\partial t} = D\left(\frac{\partial^2 x}{\partial r^2} + \frac{2}{r}\cdot\frac{\partial x}{\partial r}\right);\ 0 \leqslant r \leqslant R(t);\ t \geqslant 0 \tag{1}$$

with initial condition

$$x(r,\ 0) = x_0(r) = 0;\ 0 \leqslant r \leqslant R_0 \tag{2}$$

and boundary condition

$$x_S = x_S[R(t),\ t]. \tag{3}$$

Standard replacement of the variable

$$v(r,\ t) = rx(r,\ t) \tag{4}$$

leads to the equation

$$\frac{\partial v}{\partial t} = D\frac{\partial^2 v}{\partial r^2} \tag{5}$$

with initial condition

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 6, pp. 978-989, June, 1977. Original article submitted May 10, 1976.

$$v(r,\ 0)=rx_0=0 \tag{6}$$

and boundary conditions

$$v_S=v\,[R(t),\ t]=R(t)\,x_S(t); \tag{7}$$

$$v(0,\ t)=0. \tag{8}$$

Equality (8) is satisfied in any process and, hence, does not reflect the specificity of the posed problem. With a more rigorous approach we would have to describe more accurately the processes occurring in the vicinity of the central point. This is not the problem here.

System (5)-(8) has a unique solution[4]. The existence of a solution, unfortunately, does not indicate that it can be obtained in the form of a formula. The literature does not contain a single solution for the case of arbitrary motion of a boundary, far less for an arbitrarily varying condition on it. Below we present an attempt at an approximate, as far as possible analytical, solution of Eqs. (5)-(8) by the method of generalized thermal potentials [4-7].

The solution of the first generalized boundary-value problem (5)-(8) with a stationary boundary (r = 0) and a movable boundary [r = R(t)] will be sought in the form of a sum of double-layer potentials V_1 and V_2:

$$v(r,\ t)=V_1+V_2. \tag{9}$$

In the considered case the system of equations for the densities of these potentials has the form [4,5,6]

$$\vartheta_1(t)-\frac{1}{2\sqrt{\pi D}}\int\limits_0^t\frac{R_0+\dot{R}\tau}{(t-\tau)^{3/2}}\exp\left[-\frac{(R_0+\dot{R}\tau)^2}{4D(t-\tau)}\right]\vartheta_2(\tau)\,d\tau=0; \tag{10}$$

$$\vartheta_2(t)-\frac{1}{2\sqrt{\pi D}}\int\limits_0^t\frac{R_0+\dot{R}t}{(t-\tau)^{3/2}}\exp\left[-\frac{(R_0+\dot{R}\tau)^2}{4D(t-\tau)}\right]\vartheta_1(\tau)\,d\tau-$$

$$-\frac{1}{2\sqrt{\pi D}}\int\limits_0^t\frac{\dot{R}}{(t-\tau)^{1/2}}\exp\left[-\frac{\dot{R}^2(t-\tau)}{4D}\right]\vartheta_2(\tau)\,d\tau=-2D\,[x_{S_0}R_0+(x_{S_0}\dot{R}+\dot{x}_SR_0)\,t]. \tag{11}$$

In system (10), (11), R(t) and x_S(t) are expanded in a series of powers of t and only the linear terms are retained; i.e.,

$$R(t)=R_0+\dot{R}t;\ x_S(t)=x_{S_0}+\dot{x}_St, \tag{12}$$

where $\dot{R}$ and $\dot{x}_S$ are the rates of change of the respective quantities; R_0 is the initial bubble radius; x_{S_0} is the vapor concentration in the bubble after its injection into the liquid; and ϑ_1 and ϑ_2 are the densities of the respective potentials.

According to [4], the system (10), (11) of integral Volterra-type equations of the second kind always has a solution. It can be sought by the method of successive approximations developed for a single equation but, by performing double iteration — by fixing the form of one of the required functions in the first equation — the form of the other required function can be obtained by successive approximations and, then, substituting the second function in the second equation and fixing its form, the first function is sought, and so on. This method, of course, is exceptionally cumbersome. We will simplify it. We estimate the orders of the quantities ϑ_1 and and ϑ_2. The order of ϑ_2 in comparison with ϑ_1, apart from the integrals, is determined by the right-hand side of Eq. (11). The integrals are less than this value since, according to the theorem of evaluation of a definite integral, they are proportional to the small time. Hence, ϑ_1 will be an order less than ϑ_2. We put $\vartheta_1 = 0$. Then, instead of (11), we obtain

$$\vartheta_2(t)-\frac{2}{\sqrt{\pi D}}\int\limits_0^t\exp\left[-\frac{\dot{R}(t-\tau)}{4D}\right]\frac{\dot{R}}{\sqrt{t-\tau}}\,\vartheta_2(\tau)\,d\tau=-2D\,[x_{S_0}R_0+(x_{S_0}\dot{R}+\dot{x}_SR_0)\,t]. \tag{13}$$

Thus, system (10), (11) reduces to the single equation (13). We introduce the variable $\xi = \dot{R}\sqrt{t-\tau}/2\sqrt{D}$ and drop the subscript "2" from ϑ_2:

$$\vartheta(v)-\frac{2}{\sqrt{\pi}}\int\limits_0^v\exp(-\xi^2)\,\vartheta(\xi)\,d\xi=-2D\left[x_{S_0}R_0+(x_{S_0}\dot{R}+\dot{x}_SR_0)\,\frac{4Dv^2}{\dot{R}^2}\right], \tag{14}$$

where $\nu = \xi|_{\tau=0} = \dot{R}\sqrt{t}/2\sqrt{D}$. We expand the exponential function in the integrand in a series of powers of ξ^2 and retain only the first two terms:

$$\vartheta(\nu) = \alpha + \gamma\nu^2 + \beta \int_0^{\nu} (1-\xi^2)\vartheta(\xi)\,d\xi, \tag{15}$$

where

$$\alpha = -2Dx_{S_0}R_0;\ \beta = \frac{2}{\sqrt{\pi}};\quad \gamma = -\frac{8D^2(x_{S_0}\dot{R} + \dot{x}_S R_0)}{\dot{R}}. \tag{16}$$

Numerical evaluations show that termination of the series for the exponential function does not lead to any additional reduction of the region of applicability of the solution. Equation (15) for any β allows the construction of a sequence of functions which converge uniformly to a solution of the equation.

Performing the successive approximations, we obtain

$$\vartheta(\nu) = \alpha + \gamma\nu^2 + \alpha\beta\left(\nu - \frac{\nu^3}{3}\right) + \alpha\beta^2\left(\frac{\nu^2}{2} - \frac{\nu^4}{4}\right) + \beta\gamma\left(\frac{\nu^3}{3} - \frac{\nu^5}{5}\right) + \gamma\beta^2\left(\frac{\nu^4}{4} - \frac{\nu^6}{6}\right) + \cdots. \tag{17}$$

In compact form, for ν small in comparison with unity,

$$\vartheta = -2Dx_{S_0}R_0\left[(x_{S_0}\dot{R} + R_0\dot{x}_S)t + x_{S_0}R_0\exp\left(\frac{\dot{R}\sqrt{t}}{2\sqrt{D}}\right)\right]. \tag{18}$$

We test the hypothesis that $\vartheta_1 \ll \vartheta_2$. Let, at $\tau = 0.5t = 0.5 \cdot 10^{-2}$ sec, radius $R(\tau) = 0.2$ cm. Then the integral in (10) is of the order of $0.1\,\vartheta_2$, i.e., ϑ_1 is, in fact, relatively small. For the vapor concentration we have

$$x(r,t) = \int_0^t \frac{\vartheta(\tau)\cos\varphi}{4\sqrt{\pi D}\,(t-\tau)^{3/2}}\exp\left\{-\frac{[R(\tau)-r]^2}{4D(t-\tau)}\right\}d\tau + \int_0^t \vartheta(\tau)\frac{-R(\tau)\cos\varphi}{4r\sqrt{\pi D}\,(t-\tau)^{3/2}}\exp\left\{-\frac{[R(\tau)-r]^2}{4D(t-\tau)}\right\}d\tau, \tag{19}$$

where $\cos\varphi = +1$ for $r < R(t)$ and $\cos\varphi = -1$ for $r > R(t)$. The second term, as in Eq. (10), is dropped. Substitution of (18) gives

$$x(r,t) = \frac{1}{2r\sqrt{\pi D}}\int_0^t \frac{R(\tau)-r}{(t-\tau)^{3/2}}\left[(x_{S_0}\dot{R} + \dot{x}_S R_0)\tau + R_0 x_{S_0}\exp\left(\frac{\dot{R}\sqrt{\tau}}{\sqrt{\pi D}}\right)\right]\exp\left\{-\frac{[R(\tau)-r]^2}{4D(t-\tau)}\right\}d\tau. \tag{20}$$

Differentiating the last expression with respect to r, we obtain

$$\frac{\partial x}{\partial r} = \frac{1}{2\sqrt{\pi D}}\int_0^t \frac{1}{(t-\tau)^{3/2}}\left[(x_{S_0}\dot{R} + \dot{x}_S R_0)\tau + x_{S_0}R_0\exp\left(\frac{\dot{R}\sqrt{\tau}}{\sqrt{\pi D}}\right)\right]\exp\left\{-\frac{[R(\tau)-r]^2}{4D(t-\tau)}\right\}\left\{\frac{[R(\tau)-r]^2}{2Dr(t-\tau)} - \frac{R(\tau)}{r^2}\right\}d\tau. \tag{21}$$

It is inconvenient to use Eqs. (20) and (21) in practice: The integrals are improper. Since the solution of the bubble-growth problem requires the formula for the vapor concentration gradient on its surface, we will derive an approximate relation for $(\partial x/\partial r)[R(t), t]$. We note that the factor in square brackets in the integrand increases when $0 \le \tau \le t$ from $R_0 x_{S_0}$ to a value a little greater than $R_0 x_{S_0}$. Having in mind the region adjoining (within) the bubble surface and taking the average value of this factor, we obtain

$$\frac{\partial x}{\partial r}[R(t), t] = \frac{1}{\sqrt{\pi D}}\frac{1}{\sqrt{t}\,R(t)}\left[R_0\dot{x}_S\frac{t}{2} + R(t)x_{S_0}\right]. \tag{22}$$

In the present approximation, as follows from (22), the concentration gradient at the boundary is approximately proportional to the concentration and inversely proportional to the square root of the time. When R = 0 this and the other relationships obtained here reduce to known relations for problems with stationary regions.

We now turn to the solution of the problem of growth of a gas—vapor bubble as a whole, i.e., as distinct from the previously adopted position we will assume the values of $x_S(t)$ and $R(t)$ are unknown and are to be determined. We consider the state of the liquid adjacent to the bubble. At the interface with the gas the liquid undergoes a phase change and cools. The volume of the evaporating liquid is small in comparison with that of the vapor and, hence, the velocity of displacement of the cavity boundary relative to its center is equal to the bubble-growth rate. The main premise for solution of the "diffusion—heat removal" problem is the requirement of balance of the heat fluxes on the bubble surface.

The temperature distribution in the liquid can be represented by the heat-conduction equation. The drawbacks of this method are known: first, the need for numerical solution of the equation for the region with the movable boundary and, secondly, the presence of a semiinfinite region of thermal influence and, hence, the need for an additional criterion for the size of the thermal layer. In actual fact, however, this layer is not always limited, but, as a rule, is very thin (if we have in mind rapid processes). Expansion of the cavity in our case is an additional factor that continuously reduces the layer thickness $\delta(t)$. We note also that expansion of the cavity reduces the smoothness of the temperature curve at the outer boundary of the thermal layer. In view of the foregoing, we use the following method: We express the temperature of the liquid in the thermal layer at each instant as a linear function of the radius r. We have

$$T(r,\ t)=T_S(t)+\varkappa(t)\,[r-R(t)];\quad R(t)\leqslant r\leqslant R(t)+\delta(t);\quad t>0, \tag{23}$$

where $T_S(t) = T[R(t),\ t]$; $\varkappa(t) = (\partial T/\partial r)(t)$ is the tangent of the angle of inclination of the temperature plot to the axis Or.

Assuming that the density ρ and heat capacity c of the liquid are constant and neglecting in the sums any δ^n in comparison with R when $n \geq 1$, we find the amount of heat in the layer and differentiate it with respect to t:

$$\frac{\partial Q}{\partial t}=\frac{\partial}{\partial t}\int\limits_{R(t)}^{R(t)+\delta(t)}4\pi r^2\rho cT(r,\ t)\,dr=4\pi\rho c\,[(\dot{T}_S-\dot{\varkappa}R-\varkappa\dot{R})\,\delta R^2+$$

$$+(T_S+\varkappa R)(2R\dot{R}\delta+\dot{\delta}R^2)+\dot{\varkappa}\delta R^3+3\varkappa\delta\dot{R}R^2+\varkappa\dot{\delta}R^3]. \tag{24}$$

Since $T[R(t),\ t] = T_\infty = T_S + \varkappa\delta$, then

$$\delta=\frac{T_\infty-T_S}{\varkappa}\ \text{and}\ \dot{\delta}=\frac{-\varkappa\dot{T}_S-\dot{\varkappa}(T_\infty-T_S)}{\varkappa^2}. \tag{25}$$

We substitute (25) in (24):

$$\frac{\partial Q}{\partial t}=4\pi\rho c\left(\dot{T}_SR^2\frac{T_\infty-2T_S}{\varkappa}-\dot{\varkappa}T_SR^2\frac{T_\infty-T_S}{\varkappa^2}+2R\dot{R}T_S\frac{T_\infty-T_S}{\varkappa}\right). \tag{26}$$

This rate is equal to the heat flux into the bubble due to the entry of vapor:

$$\frac{\partial Q}{\partial t}=4\pi R^2\dot{R}x_Sh\rho_\Sigma+4\pi R^2h\rho_\Sigma D\frac{\partial x}{\partial r}\,[R(t),\ t]. \tag{27}$$

Here ρ_Σ is the molar density of the mixture of gas and vapor, and h is the latent heat of vaporization (the two quantities are constant when the temperature changes are not too great). Relation (26)-(27) is the first equation for the thermal layer (on its inner boundary).

The outer boundary of the layer is the front of the heat wave propagating through the liquid. On this front the temperature is continuous, but its gradient changes in a jump from 0 to $\varkappa(t)$. The difference in velocities of the liquid at distance $\delta(t)$ can be neglected, so that the velocity of the heat wave relative to the liquid is $\dot{\delta}$. The equality of the heat fluxes meeting the front and behind it gives a second equation

$$4\pi(R+\delta)^2\rho cT_\infty\dot{\delta}=4\pi(R+\delta)^2\lambda\varkappa, \tag{28}$$

where $a = \lambda/\rho c$ is the thermal diffusivity.

We assume that in the thermal process, as in the diffusion process, the important factor is the velocity, and not the acceleration; i.e., at least in some intervals of time the rate of change of the liquid temperature at the bubble surface T_S is approximately constant. Then from (28) we have the expression

$$\dot{\delta}=\frac{a(T_\infty-T_S)}{\delta T_\infty}, \tag{29}$$

which can be regarded as an ordinary differential equation with constant coefficients for $\delta(t)$. Its general solution is

$$\delta(t)=\sqrt{k-\frac{a\dot{T}_St^2}{T_\infty}}. \tag{30}$$

If the initial conditions at $t = t_0$ have the form $\delta = \delta_0$, $T_\infty - T_{S_0} = \Delta$, then

$$\delta=\sqrt{\delta_0^2+\frac{a\dot{T}_S}{T_\infty}\left[\left(t_0-\frac{\Delta}{\dot{T}_S}\right)^2-\left(t-\frac{\Delta}{\dot{T}_S}\right)^2\right]}. \tag{31}$$

The increase or reduction of δ depends on the sign of $\dot{T}_S$. In our case, when $t = 0$, $\delta = 0$, and $\Delta = 0$, $k = 0$ and

$$\delta=\sqrt{\frac{-a\dot{T}_S}{T_\infty}}\,t. \tag{32}$$

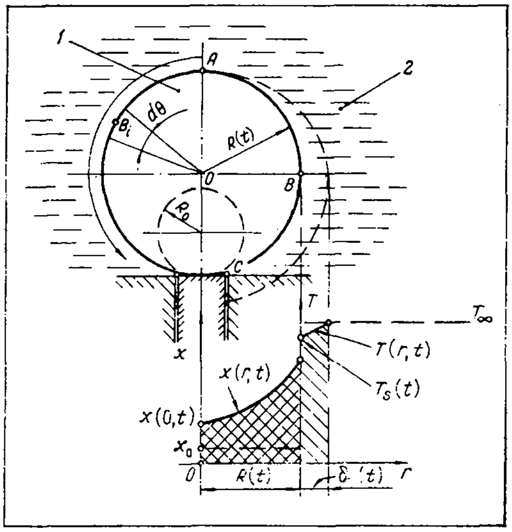

Fig. 1. Diagram of gas — vapor bubble [1) gas consisting of components A and B; 2) liquid component A].

At the initial stage of bubble growth T_S decreases, while δ increases linearly with time. Knowing $\dot{T}_S$ and δ, we find

$$\varkappa(t) = \frac{T_\infty - T_S(t)}{\delta(t)}, \tag{33}$$

i.e., the distribution of the liquid temperature.

The condition of conservation of the mass of gas in the bubble can be written in the form

$$4\pi R^2\dot{R} - 4\pi R^2\dot{R}x_S = 4\pi R^2 D\frac{\partial x}{\partial r}[R(t),\ t]$$

or, including (22),

$$\dot{R}(1 - x_S) = \frac{D}{\sqrt{\pi D}\sqrt{t}R}\left(R_0\dot{x}_S\frac{t}{2} + Rx_{S_0}\right). \tag{34}$$

The condition for thermodynamic phase equilibrium at the bubble surface is

$$x_S = \left(\frac{T_S - T_B}{T_A - T_B}\right)^m, \tag{35}$$

where m is a parameter characteristic of the particular pair of components (gas and liquid); T_A and T_B are the corresponding saturation temperatures.

Combining (28), (22), (26), (25), and (27), we obtain

$$\frac{\rho c}{\rho_\Sigma h}\left[(2\dot{R}T_S + R\dot{T}_S)\delta + \frac{aT_{S_0}R\varkappa}{T_\infty}\right] = R\dot{R}x_S + \frac{D}{\sqrt{\pi D}\sqrt{t}}\left(R_0\dot{x}_S\frac{t}{2} + Rx_{S_0}\right). \tag{36}$$

The quantities and their derivatives are connected in the usual way:

$$R = R_0 + \int_0^t \dot{R}dt;\quad T_S = T_{S_0} + \int_0^t \dot{T}_S dt \text{ and so on.} \tag{37}$$

Relations (32)-(37) form a complete system of equations for the required functions of time R, x_S, T_S, δ, and $\varkappa$. We can postulate that this system has a solution and that it is unique. It should obviously be sought by numerical methods. Before doing this, however, we must take full account of the physical scheme of the process, the actual conditions encountered in practice and in control experiments.

In all the papers that we know, the solution of the problem of growth of a gas — vapor bubble relates to a centrally symmetric situation or an axisymmetric one, which allows consideration of the effect of the flow of the liquid over the bubble. Heat and mass transfer, however, do not proceed in accordance with a spherically

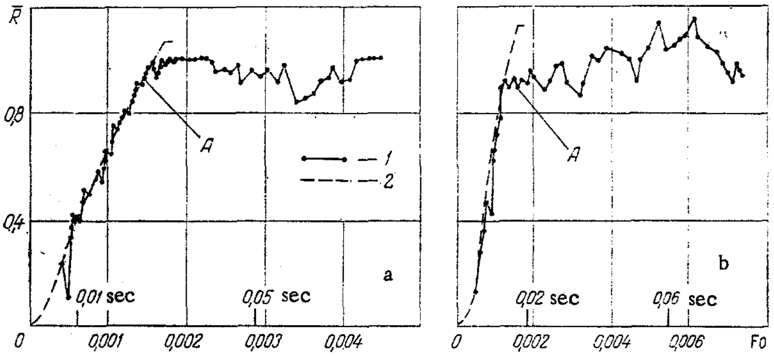

Fig. 2. Growth of air bubble in water [1) our experimental data; 2) calculation; A is the point of breakaway of the bubble from the injector]: a) $T_\infty = 94.2°C$, $x_\infty = 0.837$, $x_0 = 0.00623$, $R_\infty = 3.1$ mm, $R_0 = 1.7$ mm, $U_{av} = 20.4$ cm/sec; b) $T_\infty = 93.7°C$, $x_\infty = 0.8214$, $x_0 = 0.00623$, $R_\infty = 2.37$ mm, $R_0 = 1.34$ mm, $U_{av} = 20.2$ cm/sec, $\bar{R} = (R/R_0 - 1)/(R_\infty/R_0 - 1)$; $Fo = at/R_0^2$.

symmetric scheme at any time: Before the start of ascent the bubble remains, although for a relatively small time, on the solid surface, which leads to a difference of conditions in the vertical direction. In addition, although the bubble may be formed extremely rapidly, the rates of transfer processes at this stage are greatest, so that the change in parameters at the injection stage cannot be neglected a priori. Thus, a complete calculation of the parameters of heat and mass transfer between liquid and gas will have to include three variants of the calculation procedure: during formation of the bubbles, during their growth on the injector surface, and at the ascent stage.

To describe the first stage we propose the following scheme (Fig. 1). A batch of gas is injected into the liquid by an injector valve with a cylindrical outlet of radius R_1 in time t_0 (this time was fixed in the experiments). Let R_0 be the radius which the bubble would have at time t_0 if there was no mass transfer during injection. We assume that the injection velocity — the velocity of the bubble center — during the whole injection process is

$$V_B = \frac{2R_0}{t_0}. \tag{38}$$

During injection the gas is in the form of a spherical segment of radius $R(t)$ above the injector surface. At points A, B, and C on the surface of this segment the conditions are different, since the liquid particles at these points at a given moment land on the bubble surface at different times. At point A the liquid is continuously renewed — particles with initial temperature T_∞ arrive on it. At point C there is a particle which came into contact with the gas at the start of injection (of this batch). The point B is level with the center of the height of the segment. Several intermediate calculation points B_i can be taken between A and C. We used one or two in the calculation — this gave sufficiently accurate results (note that we talk about "points" only for convenience: for each point there is a "circle-level" at which the liquid particles have the same parameters). At the point B there arrive particles which have been involved for some time t_{B_0} in the heat- and mass-transfer process: $0 \leq t_B \leq t_{B_0}$. We take $t_{B_0} = t_0/2$.

The difference in mass-transfer conditions leads to different local rates of bubble growth. The internal pressure and surface tension, however, even out the shape of the bubble at a speed well in excess of $\dot{R}$, so that its shape is initially a spherical segment and then a sphere. The height of the segment $y(t)$, if $V_B \gg \dot{R}$, is

$$y(t) = V_B t. \tag{39}$$

The area of the bubble surface is

$$S(t) = 2\pi R(t)\, y(t). \tag{40}$$

Injection is accompanied by an increase in the diameter of the bubble. Since mass transfer at point C is very slow (and at point A, rapid), the local growth rate at point A ($\dot{R}_A$) is, consequently, the rate of increase of the vertical dimension of the bubble (and when $t > t_0$, the rate of increase of its diameter). To avoid the numerous iterations of the parameters at points A, B, and C for each instant, we found it convenient to take the average growth rate equal to

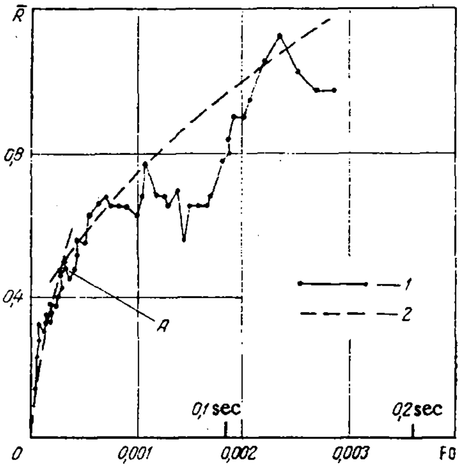

Fig. 3. Growth of nitrogen bubble in water [1) experimental data [2]; 2) calculation; A is the point of breakaway of the bubble from the injector]: $T_\infty = 94.6°C$, $x_\infty = 0.856$, $x_0 = 0.003$, $R_\infty = 5.73$ mm, $R_0 = 3.0$ mm, $V_{av} = 20$ cm/sec, $\bar{R} = (R/R_0 - 1)/(R_\infty/R_0 - 1)$; $Fo = at/R_0^2$.

$$\dot{R} = \frac{\dot{R}_A}{2} \tag{41}$$

and varying (if the inertia of the liquid is taken into account) from zero to a value $\dot{R}_{A_0}$ (at time t_0) according to a linear law, so that the bubble radius varies with time in the following way:

$$R(t) = R_0 + \frac{\dot{R}_{A_0} t^2}{4t_0}. \tag{42}$$

We also assume that at the injection stage x_s is constant and equal to some average value $x_{s_{av}}$ (to be determined).

Putting $\dot{x}_s = 0$, we obtain from (34)

$$\dot{R}_{A_0} = \frac{D x_{s_0}}{(1 - x_{s_0})\sqrt{\pi D t_0}}. \tag{43}$$

Using (40), (42), and (43), we calculate the mass of the vapor in the bubble at time t_0:

$$m_0 = 4R_0 x_{s_{cp}} \rho_\Sigma \sqrt{\pi D}\left[\frac{R_0 x_{s_0} t_0^{1/2}}{6(1 - x_{s_0})} + \frac{2R_0 t_0^{1/2}}{3} + \frac{\dot{R}_{A_0} x_{s_0}}{40(1 - x_{s_0})} + \frac{\dot{R}_{A_0} t_0^{3/2}}{14}\right]. \tag{44}$$

We now find the temperature of the liquid at points B and C. The coordinates z_B and z_C (Fig. 1) vary according to the law

$$z_B(t) = \frac{2R_0}{t_0}\left(t - \frac{t_0}{2}\right); \quad z_C(t) = \begin{cases} \dfrac{2R_0 t}{t_0} & \text{for } 0 \leqslant t \leqslant \dfrac{t_0}{2}; \\ \dfrac{2R_0}{t_0}(t_0 - t) & \text{for } \dfrac{t_0}{2} \leqslant t \leqslant t_0. \end{cases} \tag{45}$$

Through the strip of surface formed by the circle of points of type B and subtended by a central angle $\Delta\Theta$ the bubble receives, in time $t_0/2 \le t \le t_0$, the following amount of heat:

$$\Delta Q_B = 2\pi\rho_\Sigma x_{s_{av}} h\Delta\Theta \int\limits_{t_0/2}^{t_0} z_B(t)\left(R_0 + \frac{\dot{R}_{A_0} t^2}{4t_0}\right)\left(\frac{\dot{R}_{A_0} t}{2t_0} + \frac{\sqrt{D}}{\sqrt{\pi t}}\right) dt. \tag{46}$$

Similarly, for the point C circle in time $0 \le t \le t_0$

$$\Delta Q_C = 2\pi\rho_\Sigma x_{s_{av}} h\Delta\Theta \left[\int\limits_0^{t_0/2} 2R_0 t R\left(\frac{\dot{R}_{A_0} t}{2t_0} + \frac{\sqrt{D}}{\sqrt{\pi t}}\right) dt + \int\limits_{t_0/2}^{t_0} \frac{2R_0}{t_0}(t_0 - t) R\left(\frac{\dot{R}_{A_0} t}{2t_0} + \frac{\sqrt{D}}{\sqrt{\pi t}}\right) dt\right]. \tag{47}$$

The quadratures of (46) and (47) are elementary, but cumbersome, and hence are not given. The amount of heat brought from the liquid adjoining points of types B and C is, accordingly,

$$\Delta Q_{B.C} = 2\pi\rho c R^2 \Delta\Theta \frac{\varkappa_{B.C}\delta^2_{B.C}}{2}. \tag{48}$$

The calculation procedure is as follows: We assign $T_S(t_0)$, from which we find $\varkappa(t_0)$, $x_S(t_0)$, and $x_{S_{av}} = [x_S(t_0) + x_{S_0}]/2$. We substitute these values in (46)-(48) and from the discrepancy we correct $T_S(t_0)$ and carry out the iterations.

The calculations showed that the bubble ($R_0 = 0.17$ cm) during the injection time $t_0 = 0.008$ sec absorbs 14% of the total amount of heat that it can absorb. For a bubble with $R_0 = 0.3$ cm this fraction is about 9% in time $t_0 = 0.002$ sec. Thus, transfer processes must be taken with account of the start of gas injection.

In solving the problem for the following two stages of bubble growth we can again use the combined axisymmetric scheme of calculation. This is justified: The spherically symmetric solution would obviously lead to an underestimated value of the growth rate. Another point that must be taken into account is that at the stage at which the bubble remains on the injector surface the liquid moves in a meridional direction due to growth of the bubble. If we assume that the distribution of the liquid temperature along the meridian from point A to point B is linear, then in time Δt at the second stage we must add to the temperature at point B the quantity

$$\Delta T = \frac{T_\infty - T_B}{\frac{\pi R}{2}} \dot{R}\Delta t, \tag{49}$$

since the vertical component of the liquid velocity relative to point B is $\dot{R}$ (here T_B is the liquid temperature at point B).

At the stage of ascent of the bubble we must again introduce this correction, but instead of $\dot{R}$ we must put the average velocity of ascent of the bubble (U_∞) into the right-hand side of (49). The vapor concentration gradient must be calculated at all points of the surface in relation to the total time of contact of this point with the liquid, since there are always some of the liquid particles at this point and diffusion is continuous.

As examples we give the results of a comparison of the theoretical and experimental pictures of growth of air (Fig. 2a,b) and nitrogen (Fig. 3) bubbles in water. In the experiments the gas bubbles were injected into the liquid by an electromagnetic valve and were photographed by an SKS-1M high-speed camera at 4000 frames/sec. Before injection the gas was dried and heated to the liquid temperature. The bubble volume was found by graphic integration. If the vapor concentration gradient in the bubble is known, its average value over the volume at each instant can be calculated. Bubble growth ceases when this quantity attains a value corresponding to saturation at temperature T_∞. The agreement of the theoretical and experimental data is quite satisfactory: The values of the final bubble radius R_∞ differ by not more than 6%, and the values of the amount of heat removed from the liquid differ by not more than 12%.

The solution of the problem of heat and mass transfer in bubble injection by the presented method is used to obtain the variation in time of the bubble radius, the mass and heat fluxes through the bubble surface, the temperature and vapor concentration on this surface, and the relative gas humidity $\varphi(t)$. When $\varphi(t)$ and U_∞ are known, the required height of the liquid layer in the planned cooling system for the obtaining of a prescribed degree of cooling can be found; i.e., the ideas presented here can be put to effective practical use.

NOTATION

x, molar vapor concentration; D, diffusion coefficient ($m^2 \cdot sec^{-1}$); ρ, molar density ($mole \cdot m^{-3}$); c, molar specific heat ($J \cdot mole^{-1} \cdot deg^{-1}$); h, molar heat of vaporization ($J \cdot mole^{-1}$); λ, thermal conductivity ($W \cdot m^{-1} \cdot deg^{-1}$); a, thermal diffusivity ($m^2 \cdot sec^{-1}$); R, bubble radius, m; δ, thickness of thermal layer, m; T, temperature (deg); t, time (sec); τ, instantaneous time (sec); U, bubble ascent velocity ($m \cdot sec^{-1}$); V_B, bubble injection velocity ($m \cdot sec^{-1}$); S, surface area, m^2; r, z, y, coordinates (m); x, temperature gradient in thermal layer ($deg \cdot m^{-1}$); v, auxiliary function; V, generalized heat potential; ϑ, heat potential density; Fo, Fourier number. Indices: 0, initial values; ∞, final values; S, conditions on bubble surface; Σ, mixture; av, average value; $\cdot$, differentiation with respect to time; A, saturation of liquid component A; B, saturation of gas component B.

LITERATURE CITED

1. W. J. Yang and J. A. Clark, J. Heat Transfer, 86, 207 (1964).
2. V. S. Arpaci, J. A. Clark, and P. S. Larsen, Proc. Roy. Soc. (London), A283 (1965).
3. N. Tokuda, W. J. Yang, and J. A. Clark, J. Heat Transfer, 90, 371 (1968).
4. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Nauka, Moscow (1966).
5. G. N. Polozhii, Equations of Mathematical Physics [in Russian], Vysshaya Shkola, Moscow (1964).
6. L. Ya. Tslaf, Variational Calculus and Integral Equations [in Russian], Nauka, Moscow (1966).
7. O. V. Tsirlin and A. A. Yushkin, Tr. Mosk. Vyssh. Tekh. Uchilishche im. N. É. Baumana, 2, No. 195 (1974).

CRITICAL DISCHARGE OF SATURATED AND SUBCOOLED WATER THROUGH CHANNELS OF DIFFERENT SHAPES

V. G. Tonkonog, G. A. Mukhachev,
B. M. Pavlov, and I. F. Murav'ev

UDC 532.57

A complex experimental investigation of the outflow of saturated and subcooled water through channels of different forms has been carried out in the pressure range of 5-20 bar; the investigation included the measurements of the discharge, static pressure curves, moisture content field of the two-phase flow, and the photographic recording of the evaporation process. Empirical equations are proposed for determining the critical discharge.

The outflow of liquid into a medium with small counterpressure leads to the formation of a very moist two-phase flow. In the general case this process occurs with the violation of the thermodynamic equilibrium and the vapor — liquid flow thus formed is characterized by a structural inhomogeneity [1,2]. All this makes it difficult to use analytical methods for determining the maximum discharge of the evaporating liquid and forces one to take recourse to experiments. In the experimental respect the most completely investigated flow is the flow of water through diaphragms and cylindrical channels and there are practically no investigations of the flow of an evaporating liquid to channels of variable cross section, as can be seen from comprehensive reviews [3,4].

We made an attempt to study the effect of the geometry of the channel on the critical phenomena in a two-phase flow and on the critical discharge. The experiments were conducted on a device which was made according to the closed scheme and is described in [5]. As the operating substance we used clean deaerated water from the main supply to the boilers of the Kazan' TÉTs-2 thermoelectric power plant. Before the experimental segments the water parameters were measured in the pressure range of 5-20 bar and temperature range of 100-200°C. The geometry of the experimental channels is shown in Fig. 1. They include Laval nozzles with opening angles of 2-30°, diverging channels with a sharp-edged entrant orifice and opening angles of 4-30°, and channels of constant cross section. In all, 23 plane channels were tested. The dimensions of some of these are given in Table 1. The static pressure distribution along the length of the experimental channels was measured with the use of a tension probe placed in the plane wall of the channel. The fields of the phase concentrations in the flow were obtained by the radiographic method described in [2] by illuminating the two-phase flow with x-ray beams.

A combined analysis of the static pressure curves, the moisture content field, and the discharge characteristics of the channels, for which the typical results of measurements are given in Figs. 2, 3, and 4, showed that the establishment of the maximum discharge is related to the formation of a zone of intense

---

Kazan' Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 6, pp. 990-994, June, 1977. Original article submitted May 26, 1976.